

Sign convention of residues in QCD sum rules

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We show that signs of pole residues $\lambda_N, \lambda_\Lambda, \lambda_\Sigma, \lambda_\Xi$ for $\frac{1}{2}^+$ octet baryons are identical in the QCD sum rule approach. To do this we compare signs of meson-baryon coupling constants $g_{KN\Lambda}, g_{KN\Sigma}, g_{\pi\Lambda\Sigma}$ and $g_{K\Lambda\Xi}$ each other.

1 Introduction

The method of QCD sum rules has proved to be a very powerful tool to extract information about hadron properties^{1,2,3}. QCD sum rule is based on a study of the following correlation function (or correlator) of interpolating fields $j_\Gamma(x)$ which are built from quark fields and have the quantum numbers of hadrons of interest:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T(j_\Gamma(x) \bar{j}_\Gamma(0)) | 0 \rangle, \quad (1)$$

where the state $|0\rangle$ is the physical nonperturbative vacuum, and T is the time ordering operator. For example, for an interpolating field of the proton one can choose^{4,2}

$$\eta_N(x) = \epsilon_{abc} [u_a(x)^T C \gamma_\mu u_b(x)] \gamma_5 \gamma^\mu d_c(x), \quad (2)$$

where u, d are the up and down quark fields, and a, b and c are color indices. T denotes the transpose in Dirac space, and C is the charge conjugation matrix. The lowest-energy contribution to the spectral function is from the nucleon pole. Its contribution can be constructed from the matrix element

$$\langle 0 | \eta_N(0) | N(q) \rangle = \lambda_N u(q), \quad (3)$$

where $|N(q)\rangle$ is a one-nucleon state with four-momentum q^μ and $u(q)$ is a Dirac spinor for the nucleon. λ_N is the coupling strength which measures the ability of the interpolating field η_N to excite the nucleon from the QCD vacuum. In fact, the above interpolating field couples to the negative parity nucleons also. Recently, the techniques to study negative parity baryons are given in Ref.⁵ and Ref.⁶, respectively. In general, the interpolating fields for octet baryons are expressed as a combination of two fields such as⁷

$$\begin{aligned} \mathcal{O}_1^{udu} &= \epsilon_{abc} [u_a(x)^T C d_b(x)] \gamma_5 u_c(x), \\ \mathcal{O}_2^{udu} &= \epsilon_{abc} [u_a(x)^T C \gamma_5 d_b(x)] u_c(x). \end{aligned} \quad (4)$$

For example, in the case of the nucleon

$$\mathcal{O}_N = \mathcal{O}_1^{udu} + t \cdot \mathcal{O}_2^{udu}, \quad (5)$$

where t is a mixing parameter. When $t = -1$, it becomes Ioffe's choice (Eq.(2)). Similarly, one can define λ_B for another baryons as in Eq.(3). The λ_B is an experimentally unknown parameter. But, in the case of the proton it is related to the proton decay amplitude^{4,8,9,10,11,12,13,14}, or it corresponds to a size of the proton in bag models¹⁵. Some other interpretations are found in Ref.^{16,17}. In the QCD sum rule approach, we take this λ_B from usual baryon sum rules, but its form is λ_B^2 . So we do not know the sign.

In the followings we propose how to determine signs of pole residues $\lambda_N, \lambda_\Lambda, \lambda_\Sigma$ and λ_Ξ for $\frac{1}{2}^+$ octet baryons. To do this we calculate several meson-baryon coupling constants and compare their signs, each other.

2 Signs of residues $\lambda_N, \lambda_\Lambda, \lambda_\Sigma, \lambda_\Xi$

Recently, using a 3-point correlation function in the QCD sum rule method we have calculated the most relevant coupling constants in kaon production processes: i.e., $g_{KN\Lambda}$ and $g_{KN\Sigma}$ ¹⁸ (A recent status on these couplings is given in Ref.¹⁹). Some other model calculations on these couplings are found in Ref.^{20,21}.

As emphasized in Ref.¹⁸, we can not predict signs of these coupling constants within our approach. The reason is that we don't know signs of the residues $\lambda_N, \lambda_\Lambda, \lambda_\Sigma$. Our results are as follows:

$$\begin{aligned} -g_{KN\Lambda} &\simeq \frac{+}{\lambda_N \lambda_\Lambda}, \\ -g_{KN\Sigma} &\simeq \frac{-}{\lambda_N \lambda_\Sigma}, \end{aligned} \quad (6)$$

where $+$ and $-$ in the righthand sides mean that the signs of numerator are $+$ and $-$, respectively. Contributions from higher order corrections and higher dimensional operators, and the transition term are usually small. Thus, they can not change the signs of these couplings. Therefore, comparing with those of de Swart's²²

$$\begin{aligned} \lambda_N \lambda_\Lambda &\simeq +sign, \\ \lambda_N \lambda_\Sigma &\simeq +sign. \end{aligned} \quad (7)$$

This means that the signs of $\lambda_N, \lambda_\Lambda$ and λ_Σ are the same. In principle, we can change the magnitudes of these couplings with varying the mixing

parameter t . However, an optimal value of the mixing parameter t which gives reliable baryon masses is very similar to that of Ioffe's choice^{7,23}. Thus, we concentrate on Ioffe's interpolating fields in our calculations. There is the other convention²⁴ which gives $-$ to both $g_{KN\Lambda}$ and $g_{KN\Sigma}$. But, in this convention our calculation becomes

$$g_{KN\Sigma} \simeq \frac{-}{\lambda_N \lambda_\Sigma}, \quad (8)$$

and $\lambda_N \lambda_\Sigma$ is $+$ again. In the followings we present calculation of $g_{\pi\Lambda\Sigma}$ and $g_{K\Lambda\Xi}$ using the same approach¹⁸. According to de Swart's convention the signs of $g_{\pi\Lambda\Sigma}$ and $g_{K\Lambda\Xi}$ are $+$ and $-$, respectively.

In the case of $g_{\pi\Lambda\Sigma}$ the sum rule, after Borel transformation, becomes

$$\begin{aligned} \lambda_\Lambda \lambda_\Sigma \frac{M_B}{M_\Sigma^2 - M_\Lambda^2} \left(e^{-M_\Lambda^2/M^2} - e^{-M_\Sigma^2/M^2} \right) g_{\pi\Lambda\Sigma} \frac{f_\pi m_\pi^2}{\sqrt{2} m_q} = \\ - \frac{2}{\sqrt{3}} \left(\frac{7}{12\pi^2} M^4 + \frac{m_s^2}{4\pi^2} M^2 - m_s \langle \bar{s}s \rangle \right) \langle \bar{q}q \rangle, \end{aligned} \quad (9)$$

and the coupling constant has the form

$$g_{\pi\Lambda\Sigma} \simeq \frac{+}{\lambda_\Lambda \lambda_\Sigma}. \quad (10)$$

It can be a consistency check of Eq.(7). We obtain

$$g_{\pi\Lambda\Sigma} \simeq 7.53 \quad (11)$$

for $\langle \bar{q}q \rangle = - (0.230 \text{ GeV})^3$ and $\langle (\alpha_s/\pi) G^2 \rangle = (0.340 \text{ GeV})^4$. A similar result was given in Ref.²⁵ which used the 2-point correlation function.

Next, in the case of $g_{K\Sigma\Xi}$ the final expression is

$$\begin{aligned} \lambda_\Sigma \lambda_\Xi \frac{M_B}{M_\Xi^2 - M_\Sigma^2} \left(e^{-M_\Sigma^2/M^2} - e^{-M_\Xi^2/M^2} \right) \sqrt{2} g_{K\Sigma\Xi} \frac{f_K m_K^2}{2 m_q} = \\ + \left(\frac{9}{10\pi^2} M^4 + \frac{7m_s^2}{5\pi^2} M^2 - \frac{6}{5} m_s \langle \bar{s}s \rangle \right) \langle \bar{q}q \rangle. \end{aligned} \quad (12)$$

The coupling constant has the following form:

$$g_{K\Sigma\Xi} \simeq \frac{-}{\lambda_\Sigma \lambda_\Xi}, \quad (13)$$

and the value of $g_{K\Sigma\Xi}$ is

$$g_{K\Sigma\Xi} = - 7.02 \quad (14)$$

for $\langle \bar{q}q \rangle = - (0.230 \text{ GeV})^3$ and $f_K = 160 \text{ MeV}$. Therefore, the signs of λ_Σ and λ_Ξ are identical comparing with de Swart's convention. It means that the signs of residues for all octet baryons λ_N , λ_Λ , λ_Σ and λ_Ξ are the same.

3 Discussion

In the previous section we mentioned two conventions for kaon-hyperon-nucleon coupling constants. As emphasized in Ref. ²² both conventions lead to the same result for the only physically meaningful sign, $g_{KN\Lambda}$ and $g_{KN\Sigma} \cdot \mu(\Sigma^\circ\Lambda)$. Here, $\mu(\Sigma^\circ\Lambda)$ is the $\Sigma^\circ - \Lambda$ transition moment. According to the convention of de Swart, this moment is given by

$$\mu(\Sigma^\circ\Lambda) = -\frac{\sqrt{3}}{2}\mu_n \simeq +sign, \quad (15)$$

where μ_n is the neutron magnetic moment. Although an absolute value of the transition moment can be measured ²⁶, one can check our result for the sign convention by calculating the moment in the QCD sum rule approach. Until now, however, there are no works which calculate the transition moment from QCD sum rules directly. Only magnetic moments for baryons have been obtained by QCD sum rules ^{27,28,29,30} and the transition moment has been obtained from those moments ²⁸.

In conclusion, the relative signs of residues for $\frac{1}{2}^+$ octet baryons are presented. Their signs are identical in the QCD sum rule approach. But, we still don't know whether they are + or -.

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